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ABSTRACT

Scope and content charts specifying units, topics, and time allotment are provided for grades 7 and 8. Sets, numeration systems, whole numbers and positive rational numbers, ratio-proportion-and-percent, and geometry are the units covered in seventh grade. In eighth grade, the units include sets, real numbers, ratio-proportion-and-percent, geometry, coordinate geometry, and statistics. The remainder of the guide briefly explains mathematical content and suggests teaching methods. See SE 016 447 for the handbook which supplements this syllabus. (DT)

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MATHEMATICS

COURSES FOR THE SEVENTH YEAR AND EIGHTH YEAR

THE UNIVERSITY OF THE STATE OF NEW YORK/THE STATE EDUCATION DEPARTMENT
BUREAU OF SECONDARY CURRICULUM DEVELOPMENT/ALBANY, NEW YORK 12224

1971

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AND

EIGHTH YEAR

1973 Reprint

The University of the State of New York
The State Education Department
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GENERAL INFORMATION

*Seventn Year Mathematics

Since some of the material in the first unit, Sets, may have been taught in earlier grades in some schools, only a review and extension may be needed in some classes. This may permit a reduction of the time allotment for this unit.

**Eighth Year Mathematics

Because a thorough understanding of basic mathematic concepts is a prerequisite to further study, a portion of the work in the eighth year consists of a review and extension of topics taught in the seventh year. These topics are listed as *review topics*, (†).

Some eighth graders in some classes may not have had any exposure to certain of these topics in the seventh year. Self-made diagnostic quizzes may be used to determine the competency level of the class. In some areas the seventh grade outline material will prove to be a valuable help. In such cases, material may be developed *in full* by the eighth grade teacher.

FOREWORD

In the fall of 1963 several New York State Schools began working with experimental courses in mathematics for the seventh and eighth grades. The courses, commonly known as 7X and 8X, were developed for the purpose of revising the mathematics program for these grades.

A committee of teachers who had been working with the 7X and 8X courses met at the State Education Department during June 28-30, 1965, to discuss the materials and make recommendations. It was generally agreed that the courses contained too much material to be taught effectively in two years. It was recommended that a number of topics be moved to those designated as "optional" topics, either because the level of difficulty made them unsuitable to be required of all students or because of time limitations.

The committee pointed out that in the 7X course there was an inadequate development of the concepts of proportion and percent and an overemphasis on the topic, Systems of Numeration. The principal deletions from the seventh grade course that were suggested concerned manipulations in various bases, historical aspects of multiplication, figurate numbers, measurement of volume, symmetry, and coordinate geometry. The deletions from the eighth grade course were the general proof of the product of unit fractions, construction of models of geometric solids, trigonometric functions, and variation. An important suggested addition to the content of the eighth grade course was work involving negative numbers, which the committee recommended be changed from optional status to part of the basic program.

On May 27, 1966, a committee met to consider a consistent and reasonable framework for seventh and eighth grade mathematics as a recommended State course of study. The committee developed the present outline, based on the 7X and 8X materials and the evaluations and recommendations resulting from their use in the classroom, endeavoring to provide good articulation and continuity with *Mathematics K-6, A proposed Program*, and *Ninth Year Mathematics, Course I - Algebra* as revised in 1965. Since optional topics are not included in this course of study, teachers are referred to the Department publication, *Mathematics 7-8 Handbook*, for a background of optional materials which teachers may wish to include as time and student ability permit. The *Handbook* also contains helpful notes on teaching techniques.

Gordon E. Van Hooft, *Director*
Division of School Supervision

SEVENTH YEAR MATHEMATICS

Scope and Content*

Unit	Topics	Time Allotment (Days)
I.	Sets	15-20
	A) Concept	
	B) Set notation	
	C) Kinds	
	1) Empty	
	2) Finite	
	3) Infinite	
	4) Equal	
	5) Equivalent	
	6) Disjoint	
	D) Universal set and subsets	
	E) Union and intersection of sets	
II.	Systems of Numeration	10-15
	A) Definition	
	B) History of numeration	
	C) Number, numeral	
	1) Tallying	
	2) One-to-one correspondence	
	3) Cardinal and ordinal numbers	
	4) Counting and natural numbers	
	D) The decimal system	
	1) Place value	
	2) Writing a numeral	
	a) Expanded notation	
	b) Positional notation	
	c) Exponential form	
	d) Scientific notation	
	E) Bases other than ten	
	1) Grouping in other bases	
	2) Place value in other bases	
	3) Writing numerals in other bases	
	a) Positional notation	
	b) Expanded notation using factors	
	c) Expanded notation using exponents	
III.	The Set of Whole Numbers	30-35
	A) Meaning of whole numbers	
	B) Basic operations with whole numbers	
	1) Binary operation	
	2) Addition and multiplication	
	C) Inverse operations with whole numbers	
	1) Subtraction	
	2) Division	
	D) Properties of whole numbers	
	1) Closure	

*See page iv

2) Commutative
3) Associative
4) Distributive
5) Identity elements

E) Rounding whole numbers

F) Mathematical sentences
1) True and false sentences
2) Open sentences
a) Definition
b) Replacement sets
c) Solution sets
d) Equality and inequality symbols
3) Problem solving

G) Rules for divisibility

H) Prime numbers and composite numbers

I) Factors
1) Prime factors
2) Complete factorization
3) Greatest common factor
4) Relatively prime numbers

J) Multiples

K) Least common multiple

L) Order of operations

IV. The Set of Positive Rationals 35-40

A) Meaning and interpretation of fractions

B) Kinds of fractions
1) Simple or common fractions
2) Proper fractions
3) Improper fractions
4) Mixed fractions
5) Unit fractions
6) Complex fractions
7) Decimal fractions

C) Operations with fractions
1) Addition and subtraction
a) Like denominators
b) Unlike denominators
2) Multiplication and division
3) Multiplicative inverse (reciprocal)

D) Properties of positive rationals
1) Closure
2) Commutative
3) Associative
4) Distributive
5) Identity elements
6) Multiplicative inverse
7) Density

E) Simplifying rationals

F) Problem solving

G) Decimal fractions
1) Terminating decimals
2) Repeating decimals

3) Rounding decimals
4) Operations on decimals
5) Problem solving

H) Scientific notation
I) Denominate numbers

V. Ratio, Proportion, and Percent 35-40

A) Definition of ratio
B) Expressing ratios
C) Proportions
1) Definition
2) Solving
a) Cross-product rule
b) Open-sentence approach
D) Percent
1) Meaning of percent
2) Expressing percents
a) Fractional notation
b) Decimal notation
3) Working with percents
a) Finding a percent of a number
b) Finding what percent one number is of another
c) Finding a number when a percent of it is given
4) Problems involving percents
a) General problems
b) Discount problems
c) Commission problems
d) Interest problems

VI. Geometry 15-20

A) Undefined terms
B) Point
C) Line
1) Segment
2) Ray
3) Curve
D) Plane
E) Angles
1) Definition
2) Classification
3) Measure
4) Pairs
a) Vertical
b) Complementary
c) Supplementary
F) Simple closed curve
G) Polygons
1) Definition
2) Triangles

a) Classification by sides
b) Classification by angles

3) Quadrilaterals
a) Parallelogram
b) Rectangle
c) Square

4) Other polygons

H) Circle
1) Definition
2) Line segments
3) Arc
4) Semicircle
5) Circle graph

I) Perimeter
1) Definition
2) Rectangle
3) Circle

J) Area
1) Definition
2) Rectangle
3) Parallelogram
4) Triangle
5) Circle

EIGHTH YEAR MATHEMATICS

Scope and Content**

Unit	Topics	Time Allotment (Days)
I. Sets		10-15
†A) Concept	†Determine class competency - review as needed	
†B) Set notation		
†C) Kinds		
1) Universal		
2) Finite		
3) Infinite		
4) Null		
5) Equal		
6) Equivalent		
7) Disjoint		
†D) Subsets and proper subsets		
†E) Union and intersection of sets		
†F) Pictorial representation		
II. The Set of Real Numbers		40-45
A) Integers		
1) Meaning		
2) Addition and multiplication		
a) Number line		
b) Additive inVerse		
3) Subtraction and division		
4) Order of operations		
5) Properties		
a) Closure		
b) Commutative		
c) Associative		
d) Distributive		
e) Identity elements		
f) Inverse element		
†6) Divisibility		
†7) Prime and composite numbers		
†8) Prime factors		
a) Complete factorization		
b) Greatest common factor		
†9) Least common multiple		
10) Linear open sentences		
B) Rationals		
1) Definition		
2) Addition and subtraction		
a) Fractions with like signs		
b) Fractions with unlike signs		
3) Multiplication and division		
a) Fractions with like signs		
b) Fractions with unlike signs		
4) Decimal fractions		

**See page iv

†5) Scientific notation
6) Properties
a) Closure
b) Commutative
c) Associative
d) Distributive
e) Identity elements
f) Inverse element
g) Density
7) Simplifying rationals
8) Verbal problems using open sentences
†C) Terminating and repeating decimals
D) Irrationals
1) Definition
2) Representation on number line
3) Square root

III. Ratio, Proportion, and Percent 25-30
†A) Ratio
1) Definition
2) Expressing ratios
†B) Proportion
1) Definition
2) Cross product rule
3) Similar triangles
C) Percent
†1) Concepts of percent
†2) Aspects of percent
3) Problem solving
†a) Interest
†b) Commission
†c) Discount
d) Percent of increase
e) Percent of decrease

IV. Geometry 40-45
†A) Undefined terms
†B) Point
1) Collinear
2) Coplanar
†C) Line
1) Segment
2) Ray
D) Lines and sets of lines
1) Curve
2) Horizontal
3) Vertical
4) Oblique
5) Intersecting
6) Parallel
7) Skew
E) Planes

†F) Angles and angle relations
    1) Definition
    2) Classification
    3) Adjacent
    4) Vertical
    5) Complementary
    6) Supplementary
 G) Perpendicular lines
 H) Congruence
 I) Parallel lines cut by transversal
†J) Simple closed figures
 K) Polygons
    †1) Definition
    †2) Diagonal
    †3) Regular
    †4) Triangles
        a) Classification by sides
        b) Classification by angles
     5) Quadrilaterals
        †a) Parallelogram
        †b) Rectangle
         c) Rhombus
        †d) Square
         e) Trapezoid
     6) Other polygons
     7) Perimeter
     8) Area
 L) Circles
    †1) Definition
    †2) Straight line segments
    †3) Arc
    †4) Semicircle
     5) Central angle
     6) Concentric circles
    †7) Circumference
    †8) Area
 M) Geometric solids
     1) Meaning
     2) Polyhedrons
     3) Prisms
     4) Cylinder
     5) Cone
     6) Pyramid
 N) Surface area
 O) Volume
     1) Definition
     2) Prisms
     3) Cylinder
     4) Pyramid
     5) Cone

V. Coordinate Geometry — 10-15
- A) Number line
  - 1) Sets of points
  - 2) Standard description
- B) Number plane and coordinate axes
  - 1) Meaning
  - 2) Identifying points
    - a) Ordered pairs
    - b) Plotting points
  - 3) Rectangle
    - a) Plotting vertices
    - b) Area
  - 4) Right triangle
    - a) Plotting vertices
    - b) Area
    - c) Pythagorean theorem
  - 5) Graphing open sentences
    - a) Equalities
    - b) Inequalities

VI. Statistics — 15-20
- A) Data and data collection
- B) Tabulation of data
  - 1) Tallying
  - 2) Frequency
  - 3) Frequency distribution
  - 4) Cumulative frequency
- C) Interpretation of data
  - 1) Histogram
  - 2) Frequency polygon
  - 3) Relative frequency
- D) Measures of central tendency
  - 1) Mean
  - 2) Median
  - 3) Mode

SEVENTH YEAR MATHEMATICS

Suggestions to Teachers

I. SETS

The term *set* is undefined but can be considered to be a collection or group of objects or concepts. These objects or concepts are called *elements* or *members* of the set. The elements of a set may be *listed* in braces, as {1,2,3,4}, which is read, "the set whose elements are 1,2,3, and 4." The same set may be *described* in words as {all counting numbers from one through four inclusive}. Also, some sets may be *graphed* on a number line. It is suggested that simple representation of sets on the number line be taught at this time.

If a set contains no elements, it is called the *empty* or *null* set. An example of the null set is the set of all green eyed monsters in this classroom. The symbols used for null set are $\emptyset$ or { }. A set that contains a definite number of elements is a *finite* set; such a set is {1,2,3,4}. *Infinite* sets contain an unending number of elements. For example, {1,2,3,4...} denotes an infinite set using three dots, which are read as "and so on indefinitely." Sets are *equal* if they contain the same elements, not necessarily in the same order, as, {a,b,c,d} = {d,b,a,c}. Sets are *equivalent* or *matching* if they contain the same number of elements. {a,b,c,d} is equivalent to {1,2,3,4}. *Disjoint* sets are sets which contain no members in common.

The largest set under consideration is the *universal* set. *Subsets* are formed when elements of the universal set are grouped in various ways. The null set is a subset of every set. Every set is a subset of itself. Subsets other than the universal set are called *proper subsets*. The subsets of {1,2} are $\emptyset$, {1,2}, {1}, and {2}.

The *union* of sets is the set containing all the elements of the sets, but listing no element more than once. The union of {1,2,3} and {3,5,6} is {1,2,3,5,6}. The *intersection* of sets is the set containing only the elements which are common to the sets. The intersection of {1,2,3} and {3,5,6} is {3}. The intersection of disjoint sets is the null set. Simple pictorial representations are helpful at this level in clarifying union and intersection of sets.

II. SYSTEMS OF NUMERATION

A *system of numeration* is a system used to name numbers. Systems of numeration developed as a result of the necessity for counting. Students may enjoy short discussions on the Egyptian, Babylonian, and Roman systems of numeration before studying the

Hindu-Arabic system. Students should be aware that other systems of numeration existed before the decimal system, but too great an emphasis should not be placed on these systems.

At this level, students should be taught to clearly distinguish between *number*, a concept; and *numeral*, a name for a number. *Tallying* may be used to keep track of things. *One-to-one-correspondence* exists if exactly one thing or object is matched with exactly one other thing or object. Four letters and four numbers are in a one-to-one correspondence as follows:

a b c d
↕ ↕ ↕ ↕
2 3 4 5

Cardinal numbers are used to tell how many; {3,4,5,6} has four elements and its cardinal number is four. *Ordinal* numbers are numbers used to name a position; four is the second number in {3,4,5,6}.

There is a lack of agreement among authorities as to whether zero is included in the natural numbers. In this course of study, we shall define the natural numbers as {1,2,3,4,...}; notice we do not include zero.

The significance of the *decimal system* lies in the concept of *place value*. Writing a numeral as a polynomial is the same as writing it as an *expanded numeral*. Written in expanded notation, the numeral 274, would be (2 x 100) + (7 x 10) + (4 x 1). The same numeral in *positional notation* is simply 274.

Factors are two or more whole numbers which when multiplied produce a product. Students should be taught the use of power notation at this point. Emphasize that in the number 10^4, the 4 is written smaller than the 10, and is written above and to the right of it. The 4 is called an *exponent*, and the 10 is the *base*. An exponent tells us how many times we use the base as a factor, therefore, 10^4 means 10 x 10 x 10 x 10. Written in *exponential form*, 274 would be $(2 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$.

Scientific notation is a method of writing numbers as a product whose first factor is a number between one and ten, and whose second factor is a power of ten. Examples of scientific notation should be kept simple, such as:

1) $30 = 3 \times 10^1$
2) $400 = 4 \times 10^2$
3) $8000 = 8 \times 10^3$

Work in bases other than ten should be introduced only to strengthen an understanding of the decimal system.

III. THE SET OF WHOLE NUMBERS

The *whole numbers* are the natural numbers and zero, {0,1,2,3,...}. The four operations on whole numbers are binary. A *binary operation* is an operation performed on only two elements at a time. The number line should be employed in working with the operations on whole numbers, particularly with addition, subtraction, and multiplication. This will aid the student later in understanding rationals. Emphasis should be placed on the fact that subtraction is the inverse operation of addition. $a - b = c$ if and only if $c + b = a$. Also, division is the inverse operation of multiplication. $\frac{a}{b} = c$ if and only if $cb = a$. Along with the use of the number line, this emphasis will aid the student in his work with the real number system.

The set of whole numbers has the following properties:

1) Closure under addition and multiplication.
2) Commutative under addition and multiplication.
3) Associative under addition and multiplication.
4) Multiplication is distributive over addition.
5) Identity element for addition and multiplication.

These properties are clarified in Unit II of the 7X syllabus on pages 85 through 92.

Since there are various ways in rounding numbers, the method of rounding whole numbers is left to the discretion of the teacher.

At this level, the definition of an *open sentence* should be simply, a mathematical sentence which contains a *variable* or *placeholder*. An open sentence is either true or false depending upon the numbers chosen to take the place of the variable. The *replacement set* is the set from which these numbers may be chosen. If there are numbers in the replacement set which make the sentence true, this set of numbers is called the *solution set*; if there are no replacements that make the sentence true, then the null set is called the solution set. The solution set is a subset of the replacement set. The student should become familiar with the following symbols:

$=$ means "is equal to"
$\neq$ means "is not equal to"
$>$ means "is greater than"
$\geq$ means "is greater than or equal to"
$<$ means "is less than"
$\leq$ means "is less than or equal to"

We shall define *divisibility* as meaning, "can be divided by an integer without a remainder." The rules which should be discussed at this time are:

1) A number is divisible by two if the last digit in the number is an even number.

2) A number is divisible by three if the sum of the digits is divisible by three.
3) A number is divisible by five if the last digit is a five or a zero.
4) A number is divisible by ten if the last digit is zero.

These rules are helpful in complete factorization.

A *prime number* is a whole number greater than one, which is divisible only by itself and one. A *composite number* is a whole number greater than one, which is not prime. The *prime factors* of a number are all the factors of the number that are also prime numbers. A number expressed as the product of primes is the *complete factorization* of that number. Although there are only two prime factors of 56, 2 and 7, the prime factorization of 56 must be stated 2 x 2 x 2 x 7.

The *greatest common factor* of a set of numbers is the largest factor that is common to the set. Two numbers are said to be *relatively prime* to each other if their greatest common factor is one.

If any number is multiplied by a whole number other than zero, the product is said to be a *multiple* of the given number. The *least common multiple* of a set of numbers is the smallest common multiple of the set.

When no symbols of grouping appear, the *order of operations* is important. The student should know that multiplication and division are to be performed first, in the order in which they appear, before addition and subtraction.

IV. THE SET OF POSITIVE RATIONALS

Although a *rational number* is defined as the quotient of two integers, $\frac{a}{b}$, where $b \neq 0$, it is not recommended that the student be given this definition at this level. Fractions may be classified as:

1) *Simple or Common Fractions* — those fractions which have integral numerators and denominators.
$$\frac{3}{4} \qquad \frac{9}{5} \qquad \frac{200}{1000}$$
2) *Proper Fractions* — fractions in which the numerator < denominator.
$$\frac{7}{8} \qquad \frac{18}{36} \qquad \frac{3}{200}$$
3) *Improper Fractions* — fractions in which the numerator ≥ denominator.
$$\frac{4}{3} \qquad \frac{6}{6} \qquad \frac{8}{3}$$

4) *Mixed Fractions* imply the addition of an integer and a fraction; thus, the mixed number $2\frac{3}{4}$ means $2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$ as an improper fraction.

5) *Unit Fractions* are simple fractions which have a numerator of one.

$$\frac{1}{2} \qquad \frac{1}{30} \qquad \frac{1}{9}$$

6) *Complex Fractions* are fractions with a fraction in the numerator or denominator or both.

$$\frac{\frac{1}{3}}{2} \qquad \frac{2}{\frac{1}{8}} \qquad \frac{\frac{2}{3}}{\frac{3}{4}}$$

7) *Decimal Fractions* are fractions whose denominators are designated by the position of the decimal point.

$$0.3 = \frac{3}{10} \qquad .06 = \frac{6}{100} \qquad 15.5 = 15\frac{5}{10}$$

We should emphasize addition and subtraction of rational numbers in horizontal form, such as:

1) $\frac{2}{7} + \frac{3}{7} = \frac{2 + 3}{7} = \frac{5}{7}$

2) $\frac{3}{5} - \frac{2}{5} = \frac{3 - 2}{5} = \frac{1}{5}$

3) $\frac{3}{4} + \frac{5}{6} = \frac{3(3) + 2(5)}{12} = \frac{9 + 10}{12} = \frac{19}{12}$

4) $\frac{3}{4} - \frac{2}{3} = \frac{3(3) - 4(2)}{12} = \frac{9 - 8}{12} = \frac{1}{12}$

If the product of any two numbers is one, each is called the *reciprocal* of the other. *Multiplicative inverse* is another name for reciprocal. We should use the horizontal form in multiplication and division of rationals, such as:

1) $\frac{1}{5}\left(\frac{2}{3}\right) = \frac{1(2)}{5(3)} = \frac{2}{15}$

2) $2\left(1\frac{2}{3}\right) = \left(\frac{2}{1}\right)\left(\frac{5}{3}\right) = \frac{2(5)}{1(3)} = \frac{10}{3}$

3) $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3}\left(\frac{2}{1}\right) = \frac{1(2)}{3(1)} = \frac{2}{3}$

4) $\frac{1}{5} \div 4 = \frac{1}{5}\left(\frac{1}{4}\right) = \frac{1(1)}{5(4)} = \frac{1}{20}$

The set of positive rationals has the following properties:

1) Closure under addition and multiplication
2) Commutative under addition and multiplication
3) Associative under addition and multiplication

4) Multiplication is distributive over addition
5) Identity element for addition and multiplication
6) Multiplicative inverse (reciprocal)
7) Density

A simplified fractional number is one in which the numerator and denominator do not contain a common factor.

Any rational number can be written as either a terminating or infinitely repeating decimal. When performing the division, if the remainder is zero, the decimal *terminates*; if the decimal does not terminate and the digits repeat in a pattern, the decimal is an *infinitely repeating decimal*. Any repeating decimal should be written with a bar over the repeating digits:

1) $\frac{1}{3} = 0.3333... = .\overline{3}$

2) $\frac{43}{99} = 0.4343... = .\overline{43}$

3) $\frac{2}{7} = 0.285714285714... = .\overline{285714}$

Any repeating decimal may be expressed as a rational number. For example, given $N = 0.\overline{43}$, then:

$$\begin{array}{rl} 100N = & 43.\overline{43} \\ N = & 0.\overline{43} \\ \hline 99N = & 43 \\ N = & \frac{43}{99} \end{array}$$

The method of rounding decimal fractions is left to the discretion of the teacher.

As defined in Unit II, *scientific notation* is a method of writing numbers as a product whose first factor is a number between one and ten, and whose second factor is a power of ten. Students should now be prepared to handle examples such as:

1) $320 = 3.2 \times 10^2$
2) $4567 = 4.567 \times 10^3$
3) $84.1 = 8.41 \times 10^1$

Students should understand *denominate numbers* and should be taught the four operations using examples involving time, distance, liquids, age, dates, etc., provided this has not been taught thoroughly in the elementary grades.

V. RATIO, PROPORTION, AND PERCENT

The comparison of two numbers by division is called a *rate* where the units are usually different, or a *ratio*, where the

units are usually alike. Ratios may be expressed in the following ways: a to b, a:b, $\frac{a}{b}$, $(b \neq 0)$.

A *proportion* is an equality of two ratios, which may be expressed as $\frac{a}{b} = \frac{c}{d}$ or a:b = c:d, and read "a is to b as c is to d". Given any proportion $\frac{a}{b} = \frac{c}{d}$ in which three of the numbers a, b, c, or d are given, the proportion should be solved by applying the cross-product rule, that if $\frac{a}{b} = \frac{c}{d}$, then ad = bc.

Simple problems should be stressed using the open sentence approach. The following are examples:

1) If a man drove 129 miles in 3 hours, how many miles would he travel in 7 hours at the same rate.

$$\frac{3}{129} = \frac{7}{n}$$

2) If a tree casts a shadow 20 feet long at a time of day when a nearby pole 9 feet high casts a shadow of 6 feet, find the height of the tree.

$$\frac{9}{6} = \frac{h}{20}$$

3) A map has a scale of 1 inch = 240 miles. Find the actual distance from Buffalo to New York if the distance on the map is $1\frac{11}{12}$ inches.

$$\frac{1}{240} = \frac{1\frac{11}{12}}{n}$$

Students should be taught to use proportions when working with percents. No mention should be made to the students of the special cases existing for percents. Use proportions to solve such examples as the following:

1) Find 75% of 400.

$$\frac{n}{400} = \frac{75}{100}$$

2) What percent of 12 is 8?

$$\frac{8}{12} = \frac{n}{100}$$

3) If 2% of a number is 20, what is the number?

$$\frac{20}{n} = \frac{2}{100}$$

The following are examples of general problems involving percent:

1) Mrs. Addison planted 110 rose bushes in her garden. Twenty percent of them were white rose bushes. How many white rose bushes were in her garden?

$$\frac{n}{110} = \frac{20}{100}$$

2) A little league baseball team won 8 games of the 15 games played. What percent of the games played did the team win?

$$\frac{8}{15} = \frac{n}{100}$$

3) Ten weather balloons were launched successfully last year. This was $83\frac{1}{3}\%$ of the total weather balloons launched. How many weather balloons were launched?

$$\frac{10}{n} = \frac{83\frac{1}{3}}{100}$$

Previous work using "factor x factor = product" could be the basis of an alternate approach to percent.

Simple uses of mathematics involving interest, discount, commission and other applications of percent should be presented by the use of proportion. The student should thoroughly understand such applications. The following definitions are stated without development:

Interest is money paid for the use of money.
Discount is the amount of reduction off the regular price.
Commission is the amount of money a salesman receives for selling, based on the value of the goods sold.

The teacher will find sufficient examples of these problem types and others in most textbooks.

VI. GEOMETRY

Geometry, like any mathematical system, begins with *undefined terms*. If the meaning of certain basic words can be accepted intuitively, then the other terms can be defined in terms of these basic words. Some undefined terms are *point*, *straight line*, and *plane*.

A *point* has no dimensions but possesses the single property of position. A curve is a set of points represented by a figure drawn without lifting the pencil point from the page. A *line* is a curve

which is a set of points extending infinitely in opposite directions, thus possessing the single property of infinite length. A *line* is determined by two points. A *line segment* is a finite portion of a line and has two endpoints. A *ray* is an infinite portion of a line and has only one endpoint.

A *plane* can be thought of as a flat surface, possessing the properties of infinite length and infinite width. Three points, not in the same straight line, determine a plane.

An *angle* is the set of all points that are contained in two rays which have a common endpoint. Angles may be classified as *acute*, *right*, *obtuse*, and *straight*. The student should become familiar with the use of the protractor, and learn to measure acute and obtuse angles. At this time, students should be able to recognize *vertical* angles and distinguish between *complementary* and *supplementary* angles.

A curve whose ending point is the same as its starting point is a closed curve. A closed curve which separates the plane into just two parts (its boundary does not intersect itself) is called a *simple closed curve*. A simple closed curve separates the plane into three sets of points; the curve, its interior, and its exterior. The interior is called a *region*.

A *polygon* is a simple closed curve formed by line segments. *Triangles* classified by their sides are scalene, isosceles, or equilateral, and classified by their angles are acute, right, obtuse, or equiangular. Students should be able to recognize and draw *parallelograms*, *rectangles*, and *squares*. The teacher should point out *other polygons* such as *pentagon*, *hexagon*, *octagon*, and *decagon*.

A *circle* is the set of all points in a plane which are at a fixed distance from a fixed point; the fixed distance is called the *radius* and the fixed point is called the *center*. A *chord* is a line segment whose endpoints are on the circle; the longest chord in a circle is called a *diameter*. An *arc* is that part of a circle which is between two points on the circle, including these two points. An arc that is half a circle is a *semicircle*. The interpretation of the *circle graph* should be explained to the students.

Perimeter is the sum of the lengths of the line segments of a polygon or the length of a closed curve. Only the perimeter of a rectangle and circumference of a circle need to be studied at this level.

Area is the number of square units contained in the region enclosed by a polygon or a closed curve. At this level, consider the areas of the rectangle, parallelogram, triangle, and circle.

EIGHTH YEAR MATHEMATICS

Suggestions to Teachers

I. SETS

The topic of sets will need thorough development in some classes, but review will be sufficient in other classes depending on how much has been taught previously.

The term *set* is undefined but can be considered to be a collection of objects or concepts which are called *elements* or *members* of the set. The symbol epsilon, $\in$, is used to indicate that a certain object is an element of a certain set. "$3 \in A$" is read "3 is an element of set A". "$5 \notin A$" is read, "5 is not an element of set A". Sets may be identified by *listing* in braces the elements of a set such as {0,1,2,3,4}. The same set may be *described in words* as {all whole numbers less than or equal to four}.

The largest set under consideration is the *universal set*; its symbol is usually $\mathcal{U}$. A *finite* set contains a definite number of elements; such a set is {a,b,c,d}. An *infinite* set contains an endless number of elements. For example, {1,2,3,4,...} is an infinite set; the three dots denote the endless pattern. The *null* set contains no members. The set of all living dogs with wings is an example of the null set, which is symbolized by { } or $\emptyset$. *Equal* sets contain the same elements, not necessarily in the same order. *Equivalent* sets are sets with the same cardinal number, so that a one-to-one correspondence exists. Sets which contain no members in common are *disjoint* sets.

When elements of the universal set are grouped in various ways to form many sets, these sets are called *subsets*. The null set is a subset of every set. Every set is a subset of itself. All subsets are *proper subsets* except the set itself. If set A is a subset of B, this can be indicated by the symbol, $A \subseteq B$. If set A is a proper subset of B, this can be indicated by the symbol, $A \subset B$.

The *union* of set A and set B is the set consisting of all the elements in both sets; no element may be listed more than once. The union of set A and set B may be indicated by the use of the symbols $A \cup B$. The *intersection* of set A and set B is the set containing all the elements common to both sets. The intersection of set A and set B may be indicated by the use of the symbols $A \cap B$. Pictorial representation should be used to clarify the union and intersection of sets.

II. THE SET OF REAL NUMBERS

By *integer* we mean any element of the set {...,-3,-2,-1,0,1,2,3,...}. The numbers to the left of zero are called the negative

integers and positive integers are to the right of zero. These two sets of numbers and zero are the integers. Any integer is greater than any integer to its left, and there is no greatest or no least integer.

The number line should be used to introduce the concepts of addition and multiplication with the integers. Enough exercises using these operations with the number line should be given to encourage the student to develop rules for these operations.

For every integer, a, there exists an integer, b, such that the sum of a and b is zero; the integer b is called the *additive inverse* or *opposite* of a. The additive inverse is helpful in explaining the subtraction of integers.

The student should use the number line in performing subtraction on the integers; the teacher may explain division by the use of the number line. Since students should already understand that division is the inverse operation of multiplication, it should be explained to them why division by zero is undefined. For the integers a, b, and c, $\frac{a}{b} = c$, if and only if, $bc = a$; therefore, $b \neq 0$.

When no symbols of grouping appear, the *order of operations* is important. The student should know that multiplication and division are to be performed first, in the order in which they appear, before addition and subtraction.

The set of integers has the following properties:

1) Closure under addition, subtraction and multiplication
2) Commutative under addition and multiplication
3) Associative under addition and multiplication
4) Distributive of multiplication over addition and subtraction
5) Identity element for addition and multiplication
6) Additive inverse

Rules of divisibility for two, three, four, five, and ten should be discussed at this time. Practice and review of the following topics are necessary:

1) Prime and composite numbers
2) Prime factors
3) Complete factorization
4) Greatest common factor
5) Least common multiple

The basic principles of algebra used in solving equalities should be taught at this time. The addition property of equality states that if the same number is added to equal numbers, the sums are equal. The multiplication property of equality states that when equal numbers are multiplied by the same number, the products are equal.

The *set of rational numbers* consists of all numbers which can be expressed as the ratio or quotient of two integers, a and b, ($b \neq 0$). Between any two rationals, there exists another rational; infinitely many rationals exist between any two rationals. This property is called *density*. There is no greatest and no least rational. Like the integers, the rationals are either positive, negative, or zero.

Addition, subtraction, and multiplication of rationals should be taught in horizontal form, such as:

1) $\left(-\frac{2}{3}\right) + \left(+\frac{3}{4}\right) = \frac{(4)(-2) + (3)(+3)}{12} = \frac{(-8) + (+9)}{12} = \frac{+1}{12} = +\frac{1}{12}$

2) $\left(-\frac{2}{3}\right) + \left(-\frac{3}{4}\right) = \frac{(4)(-2) + (3)(-3)}{12} = \frac{(-8) + (-9)}{12} = \frac{-17}{12} = -\frac{17}{12}$

3) $\left(-\frac{2}{3}\right) - \left(+\frac{3}{4}\right) = \frac{(4)(-2) - (3)(+3)}{12} = \frac{(-8) - (+9)}{12} = \frac{(-8) + (-9)}{12} = \frac{-17}{12} = -\frac{17}{12}$

4) $\left(-\frac{2}{3}\right) - \left(-\frac{3}{4}\right) = \frac{(4)(-2) - (3)(-3)}{12} = \frac{(-8) - (-9)}{12} = \frac{(-8) + (+9)}{12} = \frac{+1}{12} = +\frac{1}{12}$

5) $\left(\frac{-3}{5}\right)\left(+\frac{4}{7}\right) = \frac{(-3)(+4)}{(5)(7)} = \frac{-12}{35} = -\frac{12}{35}$

6) $\left(\frac{-3}{5}\right)\left(-\frac{4}{7}\right) = \frac{(-3)(-4)}{(5)(7)} = \frac{+12}{35} = \frac{12}{35}$

The division of rationals should be reviewed with emphasis on the properties of multiplicative inverse and identity. For example,

$$\left(-\frac{3}{5}\right) \div \left(+\frac{4}{7}\right) = \frac{\left(-\frac{3}{5}\right)}{\left(+\frac{4}{7}\right)} = \frac{\left(-\frac{3}{5}\right)}{\left(+\frac{4}{7}\right)} \times \frac{\left(+\frac{7}{4}\right)}{\left(+\frac{7}{4}\right)} = \frac{\left(-\frac{3}{5}\right)\times\left(+\frac{7}{4}\right)}{1} = \left(-\frac{3}{5}\right)\times\left(+\frac{7}{4}\right) = \frac{(-3)(+7)}{(5)(4)} = \frac{-21}{20}.$$

This leads to: $\left(-\frac{3}{5}\right) \div \left(+\frac{4}{7}\right) = \left(-\frac{3}{5}\right) \times \left(+\frac{7}{4}\right) = \frac{(-3)\times(+7)}{(5)\times(4)} = \frac{-21}{20} = -\frac{21}{20}$

In this unit, the student needs review and practice with the four operations on *decimal fractions*, particularly with multiplication and division.

Scientific notation is a method of writing numbers as a product, whose first factor is a number between one and ten, and whose second factor is a power of ten. Practice should be given to the students using examples such as the following:

1) $6732 = 6.732 \times 10^{3}$
2) $348.51 = 3.4851 \times 10^{2}$
3) $.35 = 3.5 \times 10^{-1}$
4) $.0023 = 2.3 \times 10^{-3}$

The set of rationals has the following properties:

1) Closure under addition, subtraction, multiplication and division (except division by zero)
2) Commutative under addition and multiplication
3) Associative under addition and multiplication
4) Multiplication is distributive over addition and subtraction
5) Identity element for addition and multiplication
6) Additive inverse
7) Multiplicative inverse
8) Density

A *Simplified fractional number* is one in which the numerator and denominator do not contain a common factor, that is, the numerator and denominator are *relatively prime*.

Any rational number can be written as either a *terminating* or *infinitely repeating* decimal. When performing the division, if the remainder is zero, the decimal *terminates*; if the decimal does not terminate and the digits repeat in a pattern, the decimal is an *infinitely repeating decimal*. Any repeating decimal should be written with a bar over the repetend:

1) $\frac{1}{9} = 0.111\ldots = .\overline{1}$

2) $\frac{23}{99} = 0.2323\ldots = .\overline{23}$

3) $\frac{1}{7} = 0.142857142857\ldots = .\overline{142857}$

Any repeating decimal may be expressed as a rational number. For example, given $N = 0.\overline{23}$, then:

$$\begin{array}{rl} 100N = & 23.\overline{23} \\ N = & 0.\overline{23} \\ \hline 99N = & 23 \\ N = & \frac{23}{99} \end{array}$$

Numbers which cannot be expressed as the quotient or ratio of two integers, or as terminating or repeating decimals are called *irrational*. Students should be shown that there are points on the number line that represent irrational numbers such as $\sqrt{2}$. *Square root* must be taught at this level. The "divide and average" method is suggested, but any accepted method may be used.

III. RATIO, PROPORTION, AND PERCENT

The comparison of two numbers by division is called a *rate* where the units are different or a *ratio* where the units are alike. Ratios may be expressed as a:b or $\frac{a}{b}$, $(b \neq 0)$, either of which is read, "a to b."

An equality of two ratios forms a *proportion*. Any proportion in the form, a:b = c:d, or $\frac{a}{b} = \frac{c}{d}$, is read, "a is to b as c is to d." If $\frac{a}{b} = \frac{c}{d}$, then ad = bc, using the *cross-product rule*, which allows a proportion to be solved.

At this time, the topic of *similar triangles* should be taught, since problems involving similar triangles can be solved easily by using proportions.

Students should review and practice with percent examples by using proportions. Examples with percentage greater than 100 could be considered.

(Ex.) If 150% of a number is 120, what is the number?

$$\frac{120}{n} = \frac{150}{100}$$

Proportions should also be used in reviewing *interest*, *commission*, and *discount* problems. This open sentence approach should also be used in teaching percent of *increase* and *decrease* problems.

IV. GEOMETRY

Like all mathematical systems, geometry begins with *undefined terms*. Some of these undefined terms are point, straight line, and plane.

A *point* has the single property of position and has no dimensions. Points contained on the same straight line are *collinear* and points in the same plane are *coplanar*.

By *line*, we mean straight line unless otherwise specified. A *line* has only the property of infinite length in opposite directions. Since every place on the line has the property of position and is considered a point, therefore, a line may be thought of as an infinite set of points. Any two points determine a line.

A *line segment* is a finite part of a line and has two endpoints. A *ray* is an infinite portion of a line and has only one endpoint. Most students have a knowledge of the meaning of *horizontal* and *vertical* lines. Oblique lines may be described as two lines

that meet neither at a right angle or a straight angle. *Intersecting* or *concurrent* lines contain a common point. *Parallel lines* are lines in the same plane, that do not intersect. *Skew lines* are lines in different planes, that do not intersect, and are not parallel.

A *plane* has the properties of infinite length and infinite width. Three non-collinear points determine a plane. *Parallel* planes are planes which do not intersect. The intersection of two planes forms a straight line.

An *angle* is the set of all points on two rays with a common endpoint. The rays are called the *sides* and the common endpoint is called the *vertex*. The students should recognize that angles are classified according to size, such as acute, right, obtuse and straight, and should review measurement of acute, obtuse, and straight angles using the protractor. *Adjacent angles* are pairs of angles which have the same endpoint and a common side between them. *Vertical angles* are the pairs of non-adjacent angles formed by two intersecting lines. *Complementary angles* are two angles whose sum is equal to a right angle. *Supplementary angles* are two angles whose sum is equal to a straight angle.

Perpendicular lines are two lines that intersect so that adjacent angles have the same measure; perpendicular lines form right angles. In order to be perpendicular to a plane, a line must be perpendicular to two different intersecting lines in that plane.

A *transversal* is a line that intersects two other lines at two distinct points. If the lines being intersected by the transversal are parallel lines, the *corresponding angles* have the same measure, the *alternate interior angles* have the same measure, and the *alternate exterior angles* have the same measure.

A curve whose ending point is the same as its starting point is a *closed curve*. A closed curve which separates a plane into just two parts (its boundary does not intersect itself) is called a *simple closed curve*. A simple closed curve separates the plane into three sets of points; the curve, its interior, and its exterior. The interior is called a *region*.

A *polygon* is a simple closed curve formed by line segments. The intersection of two sides of a polygon is a vertex and the line segment joining two non-adjacent vertices is a *diagonal*. A *regular polygon* is a polygon whose sides are equal in length and whose angles have the same measure.

A *triangle* is a polygon with three sides. Students should be familiar with the classification of triangles by sides and by angles. Triangles classified by their sides are scalene, isosceles, or equilateral, and classified by their angles are acute, obtuse, right and equiangular.

A *quadrilateral* is a polygon with four sides. A *trapezoid* is a quadrilateral with only one pair of opposite sides parallel. A *parallelogram* is a quadrilateral which has both pairs of opposite sides parallel. A *rhombus* is a parallelogram which has two adjacent sides equal. A *rectangle* is a parallelogram with four right angles. The diagonals of a rectangle are equal in length. A *square* is a rectangle which has two adjacent sides equal; a square may also be defined as a rhombus with one right angle. The students should be familiar with *other polygons* such as pentagon, hexagon, septagon, octagon, nonagon, and decagon.

The students should know the meaning of and be able to work with the perimeter of polygons. Areas of the rectangle, parallelogram, and triangle should be reviewed, and the formula for the area of a trapezoid should be developed.

A *circle* is a set of all points in a plane which are at a fixed distance from a fixed point; the fixed distance is called the *radius* and the fixed point is the *center*. A *chord* is a line segment whose endpoints are on the circle, a chord passing through the center of the circle is called a *diameter*. An *arc* is that part of a circle which is between two points on the circle, including those two points. An arc that is half of a circle is a *semicircle*. A *central angle* is an angle whose vertex is the center of the circle and whose sides are radii. *Concentric circles* are circles in the same plane which have the same center but different radii. The teacher should review the topics, *circumference* and *area* of a circle.

A *geometric solid* is a closed surface which completely encloses a portion of space. A *polyhedron* is a geometric solid, consisting of nothing but flat surfaces, which are polygons. These flat surfaces are called *faces*, and the intersection of two faces is called an *edge*. Polyhedrons are classified according to the number of their faces, such as pentahedron, hexahedron, octahedron, etc. The five *regular polyhedrons* are hexahedron (a cube), tetrahedron (a pyramid), octahedron, dodecahedron, and icosahedron. A *prism* is a polyhedron with two congruent and parallel faces, and whose other faces are parallelograms. Students should be able to recognize *cylinders*, *cones*, and *pyramids*.

The *surface area* of a polyhedron is the sum of the areas of all its faces. Students should be introduced to this concept, but at this level, only the surface area of a *rectangular prism* need be considered.

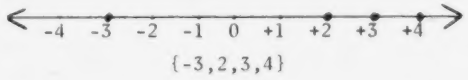
The *volume* of a geometric solid is the number of unit cubes which can be contained in a space bounded by the solid. In presenting the topic of the volume of a *prism*, first develop the formula for the volume of a rectangular prism, $V = lwh$. This will lead the student to an understanding of the formula for the volume of any prism, $V = Bh$, where B stands for the area of the base. In using the formula $V = Bh$ to determine the volume of a right *circular cylinder*, use the fact that B is equal to πr^2; this is also true

for the volume of a *circular cone*, $V = \frac{1}{3}Bh$. The equation for the volume of a *pyramid*, $V = \frac{1}{3}Bh$ should also be developed.

V. COORDINATE GEOMETRY

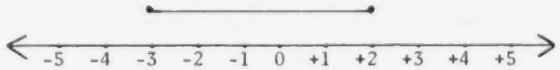
Coordinate geometry is the foundation for further studies in mathematics, as much of college mathematics from calculus on employs geometric material in analytic form. At this level, the study of coordinate geometry should involve the use of only the integers and rationals.

The students at this level are familiar with the use of the number line to represent the integers. A particular set of numbers such as {-3,2,3,4}, may be graphed as shown below.

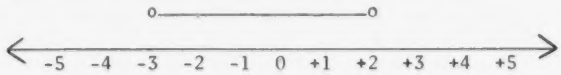


{-3,2,3,4}

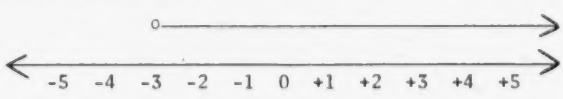
Certain infinite sets cannot be described by a process of enumeration within braces, and so a different notation is necessary. The set of all points greater than or equal to -3 and less than or equal to +2 can be indicated by $\{x \mid -3 \leq x \leq +2\}$, and is read "the set of all x such that x is greater than or equal to -3 and less than or equal to +2". This notation is called a *standard description* of a set. To graph the set described, a dot is placed above each of the endpoints -3 and +2 and these dots are connected by a line segment.



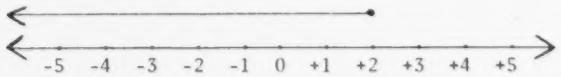
Sometimes the endpoints are not included in the set. The set described as $\{x \mid -3 < x < +2\}$, does not include the points -3 and +2. In graphing the set, small circles are placed above the points -3 and +2, and they are connected by a line segment.



A ray may be used to identify, or graph a set of points. The set described as $\{x \mid x > -3\}$, may be graphed by placing a small circle above the point -3, and drawing a ray to the right of that circle parallel to the number line.



The set described as $\{x \mid x \leq +2\}$, would be graphed by placing a dot above the point +2 and drawing a ray from the dot toward the left, parallel to the number line.



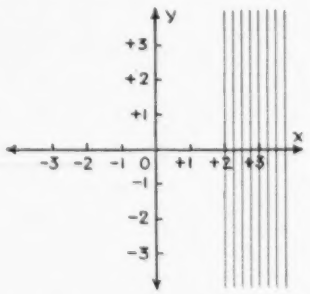
The type of *number plane* to be considered in this unit is the *coordinate plane* which is identified by the use of two number lines intersecting at right angles. It is important to stress that the same unit of distance is to be used on each of these number lines. The line drawn from left to right is called the *x-axis*. The other line is called the *y-axis*. The point of intersection of the two axes is the zero point, or *origin*. In the number plane, each point of the plane is associated with an *ordered pair* of real numbers, (x,y), called *coordinates* of the point. The first number of the pair tells the directed distance the given point is from the origin along the x-axis. The second number of the pair tells the directed distance the given point is from the origin along the y-axis.

The student should be given experience in *plotting points* identified by ordered number pairs. Points may be chosen, so that by connecting the consecutive points with straight line segments, *rectangles* may be identified. Care should be taken in choosing vertices which will make the sides of the rectangle parallel to the axes, so that *area* may also be considered.

The vertices of a *right triangle* should be plotted, so that the base and altitude are parallel to the axes; the *area* of the resulting triangle can be found. Only simplified use of the *Pythagorean theorem* should be considered in order to avoid the use of irrational numbers.

The students should be given practice in graphing *open sentences* which result in lines parallel to the x-axis or y-axis. The x-axis is the line containing the set of all points represented by the equality $y = 0$; the y-axis is represented by $x = 0$. Simple *equalities* in two variables such as $x = y$, $x = 2y$, and $\frac{1}{2}x = y$, should also be graphed at this time.

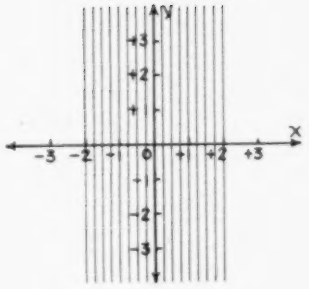
Students may be given exercises in graphing *inequalities* in which parts of planes are bounded by one or two lines.



The set of points indicated by the inequality $x \geq +2$ consists of all

points two or more units to the right of the y-axis. This is graphed on the previous page.

The set of points indicated by the inequality $-2 \leq x \leq +2$ consists of all points two units to the left of the y-axis, the y-axis, and all points two units to the right of the y-axis.



VI. STATISTICS

In this unit, students investigate elementary aspects of *statistics*, in particular, fundamental concepts associated with the organization and presentation of *data* in a form that makes a meaningful analysis of it possible.

There are several ways of *tabulating data*. One way is to list in some type of order all the different numbers that occur in the data and then show how often each score occurred by the use of *tally* marks. The number of times each number appears is called the *frequency* of that number. This type of tabulation of data is called a *frequency distribution*. *Cumulative frequency* shows, for any given score, the total frequency of all scores up to and including that score.

A *histogram* is a particular kind of vertical bar graph showing the frequency distribution of a given set of data. A *frequency polygon* is a broken line graph representing the frequency distribution of a given set of data.

The illustrations on the next page demonstrate a method for setting up score, tally, frequency, graphing a histogram, and a frequency polygon.

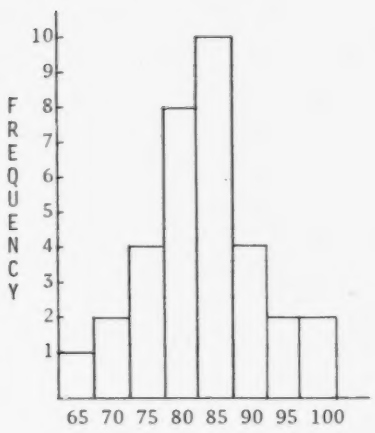
Relative frequency is the percent of the total number of scores that a frequency represents. Decimal numerals are used on a vertical line at the right of a histogram or frequency polygon, to illustrate relative frequency. At this level students should learn to construct and *interpret* histograms and frequency polygons.

Central tendency, or that part of the distribution at which many of the scores are concentrated, is measured by some sort of an average, of which there are several kinds. The *arithmetic mean* of a set of numbers is found by dividing the sum of the given numbers by the cardinal number of the given set. The *median* of a set of numbers is the middle item when the items have been arranged in order of size. If there is no middle one, then the arithmetic mean of the two

middle ones is taken as the median. The number that occurs most frequently in a set is called the *mode*. When no number occurs more than once the mode does not exist; if two or more numbers occur with the same frequency, then each of the numbers is a mode.

Score	Tally	Frequency
100	//	2
95	//	2
90	////	4
85	~~////~~ ~~////~~	10
80	~~////~~ ///	8
75	////	4
70	//	2
65	/	1

HISTOGRAM



FREQUENCY POLYGON